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TRANSFER OF HEAT AND MOISTURE IN FREEZING OF GROUNDS

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Approximate (consistent with experiments) methods of solution of problems of freezing of moist grounds of semiinfinite and finite length with different boundary conditions have been proposed. Methods of obtaining the parameters of transfer of heat and moisture from experiments have been given. The analytical and experimental results obtained have been discussed.

Much attention is given to the problem of heat and mass transfer in grounds at negative temperatures. Physical and mathematical models and methods of different complexity have been proposed for solution of problems with a moving boundary [1–10]. The use of complex mathematical models is justified where there is a data bank of coefficients involved in the system of heat- and mass-transfer equations and their dependences on the temperature, moisture content, and structure of the material. The transfer coefficients must be obtained under conditions identical to those taken in calculations of the transfer of heat and moisture in grounds. The reliability of calculation will be low in the absence of reliable information on transfer parameters for any mathematical models. Simple models with a minimum number of coefficients involved in them are preferable for prediction calculations of heat and mass transfer in grounds.

Experimental investigations have been carried out with peaty ground. Any ground receives (or gives up) heat from air and in the form of solar radiation. In winter, the radiation flux is small, whereas in the spring period melting of the ground mainly depends on the inflow of solar radiation. In the presence of a snow cover, one must take into account its "thermal resistance" [11].

In developing physical and mathematical models, the authors proceeded from the following experimental results and assumptions based on them.

1. Freezing processes in grounds are extended. The movement of the phase boundary is approximated by the dependence $\xi = \beta\sqrt{\tau}$. Its derivative (velocity), as follows from the experiments, is small ($d\xi/d\tau \sim 10^{-7}$ m/sec). In this connection, the perturbed fields of the temperature T and the moisture content u in the ground relax to the steady-state distributions of T and u more rapidly than the boundaries of the interval at which the specified boundary conditions are thus satisfied move. Therefore, the process of freezing (melting) can be considered as a successive change of steady states for which the temperature and moisture-content distributions in the ground layer of thickness $y(\tau)$ at the instant of time τ turn out to be asymptotically similar to these quantities in the layer of constant thickness $y = y(\tau)$. Such an approach enables us to obtain simplified solutions of problems with moving boundaries [2–4] by setting $y'(\tau)$ and the derivatives of higher order equal to zero in them.

2. The thermal transfer of moisture can be disregarded at the temperatures of phase transition of water to ice [12, 13]. The temperate field in the sample of the material becomes steady-state 20 to 40 times faster than the moisture-content field [11–13]. Therefore, the transfers of heat and moisture should be considered separately.

3. The coefficients of a mathematical model are variable quantities in the process of the transfer itself because of the change in the temperature, moisture content, and structure of the ground (in connection with the shrinkage in the melt zone and swelling in the frozen zone). This necessitates methods of obtaining the average coefficients (criteria) of heat and mass transfer over the entire period of freezing (melting) of the ground.

4. The temperatures, solar-radiation intensities, precipitation, and other characteristics of external heat and mass exchange of the ground with the atmosphere, which vary with the time of day and with the season, must be averaged for each step of calculation [11].

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On the basis of these assumptions, we apply to the equation

$$\frac{\partial w}{\partial \tau} = a(x)\frac{\partial^2 w}{\partial x^2} + b(x)\frac{\partial w}{\partial x} + c(x)\,w + f(x, \tau)\,, \quad y_1(\tau) < x < y_2(\tau)\,, \quad \left| y_2(\tau) - y_1(\tau) \right| > 0\,, \tag{1}$$

the finite integral transformation with the kernel [14]

$$K(x, \rho) = \sigma(x)\,N(x, \rho)/M(\rho)\,. \tag{2}$$

After the integration within the above limits, we obtain

$$\int_{y_1}^{y_2} \frac{\partial}{\partial \tau} w(x, \tau)\,K(x, \rho)\,dx = \int_{y_1}^{y_2} \left\{ \frac{\partial^2}{\partial x^2}[a(x)\,K(x, \rho)] - \right.$$

$$\left. - \frac{\partial}{\partial x}[b(x)\,K(x, \rho)] + c(x)\,K(x, \rho) \right\} w(x, \tau)\,dx + H(x, \rho) + F(\rho, \tau)\,, \tag{3}$$

$$H(x, \rho) = \left\{ \left[a(x)\frac{\partial w(x, \tau)}{\partial x} + b(x)\,w(x, \tau) \right] K(x, \rho) - w(x, \tau)\frac{\partial}{\partial x}[a(x)\,K(x, \rho)] \right\} \Bigg|_{x=y_1}^{x=y_2}.$$

According to the rule of differentiation of a certain integral with respect to the parameter τ, we have

$$\int_{y_1}^{y_2} \frac{\partial}{\partial \tau} w(x, \tau)\,K(x, \rho)\,dx = \frac{\partial}{\partial \tau}\int_{y_1}^{y_2} w(x, \tau)\,K(x, \rho)\,dx - w(x, \tau)\;K(x, \rho)\frac{\partial y}{\partial \tau}\Bigg|_{y_1}^{y_2} - \int_{y_1}^{y_2} w(x, \tau)\frac{\partial K(x, \rho)}{\partial \tau}\,dx\,.$$

The derivative $\partial K(x, \rho)/\partial \tau$ can be represented in the form

$$\frac{\partial K(x, \rho)}{\partial \tau} = \frac{\partial K(x, \rho)}{\partial x}\frac{dx}{d\tau} + \frac{\partial K(x, \rho)}{\partial \rho}\frac{\partial \rho}{\partial y}\frac{dy}{d\tau}\,.$$

Not only can the boundaries $y_i(\tau)$ move in freezing but also any "points" of a body, since the dimensions of the ground layers change due to the swelling in the frozen layer and shrinkage in the melt layer [2]. The eigenvalues ρ depend on the interval $[y_2(\tau) - y_1(\tau)]$. For higher velocities of movement of the boundaries and points of a body $y_i'(\tau)$ and $x'(\tau)$, we have $\partial K(x, \rho)/\partial \tau \neq 0$.

Since the process of freezing occurs with a low rate ($y_i'(\tau)$, $x'(\tau) \to 0$), we can take

$$\int_{y_1}^{y_2} \frac{\partial}{\partial \tau} w(x, \tau)\,K(x, \rho)\,dx \approx \frac{\partial}{\partial \tau}\int_{y_1}^{y_2} w(x, \tau)\,K(x, \rho)\,dx\,.$$

The kernel of the transformation $K(x, \tau)$ is found from the equation

$$\frac{\partial^2}{\partial x^2}[a(x)\,K(x, \rho)] - \frac{\partial}{\partial x}[b(x)\,K(x, \rho)] + [c(x) + \rho^2]\,K(x, \rho) = 0\,. \tag{4}$$

By setting

$$\frac{d}{dx}[a(x)\,\sigma(x)] = b(x)\,\sigma(x)\,, \tag{5}$$

we find, within a constant, the value of

$$\sigma(x) = \exp\left\{-\int \frac{1}{a(x)}\left[\frac{da(x)}{dx} - b(x)\right]dx\right\}. \tag{6}$$

Having substituted expressions (2) and (5) into (4), we obtain the Sturm–Liouville equation

$$\frac{d}{dx}\left[p(x)\,\frac{dN(x,\rho)}{dx}\right] - [q(x) - \rho^2\sigma(x)]\,N(x,\rho) = 0\,. \tag{7}$$

Having used relations (2) and (5), we transform the boundary function $H(x, \rho)$:

$$H(x,\rho) = \frac{p(x)}{M(\rho)}\left[N(x,\rho)\,\frac{\partial w(x,\rho)}{\partial x} - \frac{\partial N(x,\rho)}{\partial x}\,w(x,\tau)\right]\Bigg|_{x=y_1}^{x=y_2}. \tag{8}$$

With account for relations (4) and (8), we rewrite Eq. (3) as follows:

$$\frac{d\overline{w}(\rho,\tau)}{d\tau} + \rho^2\overline{w}(\rho,\tau) = H_j(x,\rho)\Bigg|_{x=y_1}^{x=y_2} + F(\rho,\tau)\,. \tag{9}$$

After the integration of (9) we obtain

$$\overline{w}(\rho,\tau) = \left[\overline{w}(\rho,0) + \int_0^\tau \exp(\rho^2\tau)\;H_j(x,\rho)\Big|_{x=y_1}^{x=y_2} d\tau + \overline{F}(\rho,\tau)\right]\exp(-\rho^2\tau)\,. \tag{10}$$

Simultaneous solution of Eqs. (7) for the functions $N(x, \rho_i)$ and $N(x, \rho_j)$, where ρ_i and ρ_j are any two numbers of the set of numbers ρ_n, and integration within the limits $[y_1, y_2]$ yield

$$\int_{y_1}^{y_2} \sigma(x)\,N(x,\rho_i)\,N(x,\rho_j)\,dx = \frac{p(x)\left[N(x,\rho_i)\,\frac{\partial N}{\partial x}(x,\rho_j) - \frac{\partial N}{\partial x}(x,\rho_j)\,N(x,\rho_j)\right]\Bigg|_{x=y_1}^{x=y_2}}{\rho_i^2 - \rho_j^2}. \tag{11}$$

The boundary conditions for the eigenfunctions $N(x, \rho_i)$ are homogeneous for any eigenvalues ρ_i; therefore, the right-hand side of Eq. (11) is equal to zero when $\rho_i \neq \rho_j$ or to $M(\rho_i)$ when $\rho_i = \rho_j$ for any values of y_1 and y_2 at $\tau \geq 0$.

The inversion formula is written as follows:

$$w(x,\tau) = \sum_{n=1}^{\infty}\left[\overline{w}(\rho_n,0) + \int_0^\tau \exp(\rho_n^2\tau)\;H(x,\rho_n)\Big|_{x=y_1}^{x=y_2} d\tau + \overline{F}(\rho_n,\tau)\right]\exp(-\rho_n^2\tau)\,N(x,\rho_n)\,. \tag{12}$$

The normalizing divisor can be represented in the form

$$M(\rho_i) = \int_{y_1}^{y_2} \sigma(x)\,N^2(x,\rho_i)\,dx = \left|\frac{p(x)}{2\rho_i}\left[\frac{\partial N(x,\rho_i)}{\partial \rho_i}\,\frac{\partial N(x,\rho_i)}{\partial x} - \frac{\partial^2 N(x,\rho_i)}{\partial x\,\partial \rho_i}\,\partial N(x,\rho_i)\right]\right|\Bigg|_{x=y_1}^{x=y_2}.$$

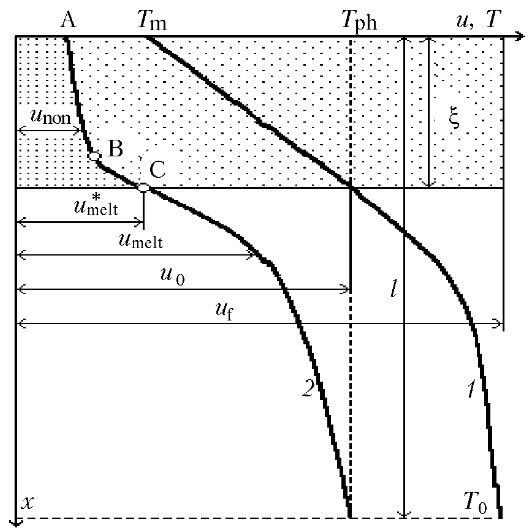

Fig. 1. Diagram of distribution of the temperature T (1) and moisture content u (2) in the frozen ground (the frost layer is hatched).

The resulting solution of (12) enables us to calculate the fields of temperatures and moisture content if the distributions of the coefficients $a(x)$, $b(x)$, and $c(x)$ over the layer depth are predetermined experimentally. These coefficients, as has been noted in description of the assumptions (item 3), depend on the phase transitions and structural changes of the ground in the process of transfer. For prediction evaluations of the heat and mass transfer in the ground layer we should set $b(x) = 0$ and $c(x) = 0$ in Eq. (1) and take the value of a to be constant and average over the layer thickness.

In solving problems with moving boundaries [2, 4], the functions $y_i(\tau)$ are considered to be known. They are found from experiment or by solution of inverse problems. Their values can be computed for any fixed time τ. The interval $[y_1, y_2]$ is taken to be constant for this time. For other values of τ the intervals will be different. In this connection, we must again determine their values and the criteria of transfer, involving the limits of the intervals, and the eigenvalues ρ_n obtained from characteristic equations. Thus, the movement of the boundaries of an interval is taken into account by the change in the values of ρ_n.

The approach proposed enables us to obtain approximate solutions of problems with moving boundaries for bodies of different configurations and with boundary conditions of the first to fourth kind. For this purpose we should use the ready solutions of such problems with fixed boundaries [1, 2].

Let us find the temperature distribution in a frozen column of ground of finite length l. We take the temperature of the ground $T_1(0, \tau) = T_m$ (Fig. 1, curve 1) at the boundary of the frozen and melt zones $T_1(\xi, \tau) = T_2(\xi, \tau) = T_{ph}$ and at the lower boundary $T_2(l, \tau)$ to be equal to the initial constant temperature T_0. In freezing, the temperature of the melt ground near the zone of phase transition is close to T_{ph}; therefore, the initial temperature of the frozen layer is taken to be equal to T_{ph}. We have the values of the functions $y_1 = 0$ and $y_2 = \xi$ for the frozen zone $0 < x < \xi$ and $y_1 = \xi$ and $y_2 = l$ for the melt zone $\xi < x < l$.

By analogy with the solutions of the problem with a fixed boundary [1], we write the solutions for any fixed values of the time τ and the limits ξ:

$$\Theta_1 = \frac{T_1(x, \tau) - T_m}{T_{ph} - T_m} = \frac{x}{\xi} + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\pi} \sin\left[n\pi\left(1 - \frac{x}{\xi}\right)\right] \exp\left[-(n\pi)^2 \frac{a_1\tau}{\xi^2}\right], \quad 0 \le x \le \xi\,; \tag{13}$$

$$\Theta_2 = \frac{T_2(x, \tau) - T_{ph}}{T_0 - T_{ph}} = \frac{x - \xi}{l - \xi} + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\pi} \sin\left[n\pi \frac{l - x}{l - \xi}\right] \exp\left[-(n\pi)^2 \frac{a_2\tau}{(l - \xi)^2}\right], \quad \xi \le x \le l\,, \tag{14}$$

and

$$\Theta_1 = \frac{T_1(x,\tau) - T_m}{T_{ph} - T_m} = \text{erf}\, z_{01} \frac{x}{\xi} - \sum_{n=1}^{\infty} \left[\text{erfc} \left[z_{01} \left(2n + \frac{x}{\xi} \right) \right] - \text{erfc} \left[z_{01} \left(2n - \frac{x}{\xi} \right) \right] \right], \quad 0 \le x \le \xi \,, \tag{15}$$

$$\Theta_2 = \frac{T_2(x,\tau) - T_{ph}}{T_0 - T_{ph}} = \text{erf} \left[z_{02} \left(\frac{x}{\xi} - 1 \right) \right] - \sum_{n=1}^{\infty} \left[\text{erfc} \left[z_{02} \left(2n \left(\frac{l}{\xi} - 1 \right) + \left(\frac{x}{\xi} - 1 \right) \right) \right] - \right.$$

$$\left. - \text{erfc} \left[z_{02} \left(2n \left(\frac{l}{\xi} - 1 \right) - \frac{x}{\xi} + 1 \right) \right] \right], \quad \xi \le x \le l \,. \tag{16}$$

The solutions (13) and (14) and (15) and (16) can conveniently be used at long and short freezing times respectively.

The coefficients (criteria) of transfer are determined from the values of the temperature $\bar{T}$, averaged over the layer thickness. Integrating Eqs. (13) and (15) from zero to ξ and Eqs. (14) and (16) from ξ to l, we obtain for the frozen ($i = 1$) and melt ($i = 2$) layers

$$\bar{\theta}_i = \frac{1}{2} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{\exp\left[-(2n-1)^2 \pi^2 \text{Fo}_i\right]}{(2n-1)^2} \,, \tag{17}$$

$$\bar{\theta}_i = 1 - 2 \sqrt{\frac{\text{Fo}_i}{\pi}} + 4 \sqrt{\text{Fo}_i} \sum_{n=1}^{\infty} (-1)^{n+1} \, \text{ierfc} \frac{n}{2 \sqrt{\text{Fo}_i}} \,. \tag{18}$$

For high values of the Fo_i numbers we can restrict ourselves to the first term of the series expansion in solution of (17) and calculate these numbers. For small Fo_i (<0.1) it will suffice to restrict ourselves to the first two terms of relation (18).

The transfer of water from the melt zone to the frozen zone is caused by the sharp decrease in the fraction of moisture in the liquid phase because of the ice formation at a temperature of 273 K or lower (Fig. 1, curve 2). For this reason, gradients of moisture content appear at the boundary of the above zones. The distribution of the part of moisture u_{non} that does not become ice depends on the temperature distribution in the frozen layer. The value of u^*_{melt} corresponds to the amount of nonfrozen moisture at the phase-transition temperature T_{ph} obtained in calorimetric experiments. For each ground there is a dependence (characteristic of it) of the amount of nonfrozen water u_{non} and u^*_{melt} on the negative Celsius temperature. The boundary of phase transition shifts deep into the ground layer with freezing but the values of u^*_{melt} remain constant, as experiments show [11]. This enables us to take boundary conditions of the first kind with a constant u^*_{melt} at the boundary of the frozen and melt zones.

The distribution of the moisture content in the semiinfinite region of values $\xi < x < \infty$ is found by solution of the moisture-transfer equation

$$\frac{\partial u_{melt}(x,\tau)}{\partial \tau} = D_{melt} \frac{\partial^2 u_{melt}(x,\tau)}{\partial x^2} \tag{19}$$

with the initial and boundary conditions

$$u_{melt}(x, 0) = u_0 \,, \tag{20}$$

$$u_{melt}(\xi, \tau) = u^*_{melt} \,, \tag{21}$$

TABLE 1. Values of the Function $f(z_0)$

z_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$f(z_0)$	6.2935	3.4869	2.5601	2.1027	1.8327	1.6561	1.5325	1.4419	1.3731	1.3195
z_0	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
$f(z_0)$	1.2420	1.1897	1.1525	1.1253	1.1045	1.0884	1.0751	1.0659	1.0576	1.0549

$$\partial u\,(\infty, \tau)/\partial x = 0\,. \tag{22}$$

Since the excess moisture migrating to the freezing front becomes ice at the boundary of the melt and frozen zones ($x = \xi$), we can write the additional condition

$$D_{\text{melt}}\,(du_{\text{melt}}/dx) = (u_{\text{f}} - u^{*}_{\text{melt}})\,d\xi/d\tau\,. \tag{23}$$

The general solution has the form

$$U_{\text{melt}} = (u_0 - u_{\text{melt}})/(u_0 - u^{*}_{\text{melt}}) = \operatorname{erfc} z/\operatorname{erfc} z_0\,. \tag{24}$$

To determine the moisture content in the frozen zone of thickness ξ we use the balance equation

$$u_{\text{f}} = u_0 + \frac{1}{\xi}\int_{\xi}^{\infty}(u_0 - u_{\text{melt}})\,dx\,. \tag{25}$$

The solution can be represented as

$$U_{\text{f}} = (u_{\text{f}} - u^{*}_{\text{melt}})/(u_0 - u^{*}_{\text{melt}}) = \exp\,(-z_0^2)/(\sqrt{\pi}\,z_0 \operatorname{erfc} z_0)\,. \tag{26}$$

Having determined the values of u_{f} and u^{*}_{melt} from the experimental data and then the simplex U_{f} from Table 1, we find the quantity z_0. If the function erfc z_0 is expanded in a series, the value of z_0 can be computed from the formula $z_0 \cong [0.5/(1-(1/U_{\text{f}}))]^{1/2}$ for $U_{\text{f}} < 1.05$.

The experiments on freezing were carried out with cylindrical columns (heat-insulated from the lateral surface) of diameter 3.5 and length 10 to 33 cm with peat [11]. Six identical samples were simultaneously frozen. The upper ends of the samples were blown with air cooled in a refrigerator. In the experiments "in the presence of moisture insulation," the samples were set with their lower ends on a metal electric heater ensuring a constant prescribed temperature of 273 to 283 K. Part of the experiments were carried out with additional feeding of water of a certain and constant temperature to the samples via their open lower ends ("in contact with water"). The temperature was measured by thermocouples accurate to ±0.1 K. The final moisture content over the height of the columns was determined by dismounting them into individual layers and determining u of the peat by the standard thermogravimetric method.

According to Fig. 2, the simplexes are $\theta_1 = 0.657$ and $\theta_2 = 0.655$. The average temperatures over the thickness of the frozen and melt layers are equal to 270.6 and 274.9 K, whereas the temperatures on the upper and lower surfaces of a sample and at the boundary of the frozen and melt zones correspond to 266, 275.9, and 273 K.

Restricting ourselves to the first term of the sum in Eq. (17), we calculate the values $\text{Fo}_1 = \pi^2 \ln[(\pi^2/4)(\theta_1 - 0.5)] = 0.096$ and $\text{Fo}_2 = 0.097$. If we disregard all the terms of the sum of Eq. (18), we obtain more exact values of the Fourier numbers for $\text{Fo}_i < 0.1$: $\text{Fo}_1 = (1-\theta_1)\pi^2/4 = 0.092$ and $\text{Fo}_2 = 0.093$. Accordingly we obtain the numbers $z_{01} = [2\sqrt{\text{Fo}_1}]^{-1} = 1.65$ and $z_{02} = [2(L-1)\sqrt{\text{Fo}_2}]^{-1} = 0.546$, $L = l/\xi = 4$, and the values of the thermal diffusivities $a_1 = [\beta/(2z_{01})^2] = 1.4\cdot10^{-7}$ m^2/sec and $a_2 = 1.27\cdot10^{-8}$ m^2/sec. The value $\beta = 1.23\cdot10^{-4}$ m/sec$^{1/2}$ necessary for calculation was experimentally determined from the angular coefficient of the linear dependence $\xi = f(\tau^{1/2})$.

From Fig. 2 (curve 2) it follows that moisture is transferred from the zone near the freezing front. Here we can apply the equations proposed for a semiinfinite medium. Since we have the values $u_{\text{f}}/u_0 = 1.036$ and $u^{*}_{\text{melt}}/u_0 =$

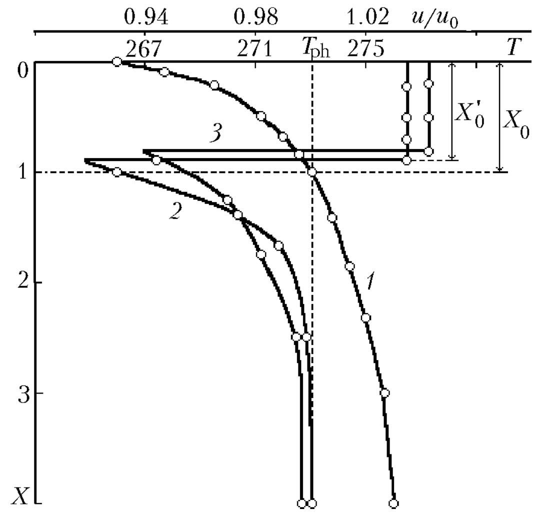

Fig. 2. Distribution of the temperature T (1), the relative moisture content u/u_0 (2), and the radioactivity R/R_0 (3) in the depth $X = x/\xi$ of the frozen and melt layers of peat ground; X_0, X_0', relative coordinates corresponding to the boundary (T = 273 K) and center of the zone of phase transition; points, experiment; curves, calculation; u_0 = 2.8 kg/kg.

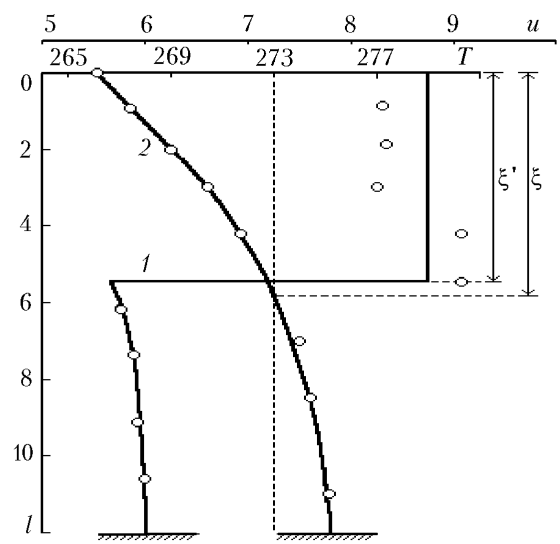

Fig. 3. Distribution of the temperature T (1) and the moisture content u (2) in a peat-filled column of length 12 cm, having heat insulation of the lower end; ξ = 5.7 and ξ' = 5.4 cm; u_0 = 7.25 kg/kg.

0.92, the simplex is $U_f = (u_f - u^*_{melt})/(u_0 - u^*_{melt}) = 1.45$. We find $z = 0.8$ from Table 1 and determine the coefficient of diffusion of moisture in the melt zone $D_{melt} = (\beta/2z_0)^2 = 0.59 \cdot 10^{-8}$ m^2/sec. The distributions of the temperature and the moisture content over the sample's length (Fig. 2), calculated from Eqs. (15), (16), and (24) based on the obtained z_{01}, z_{02}, and z_0, are consistent with experiment. To the temperature distribution in the columns of a comparatively large length l (when $\xi << 1$) we can also apply the Stefan equations [1] for freezing of grounds.

The temperature of the lower end of the column insulated at the bottom (Fig. 3) was constant. Therefore, we can apply the above method of calculation to determination of the numbers and coefficients of transfer of heat.

At an average temperature of the frozen and melt zones of 270 and 274.5 K, the simplexes are respectively equal to $\bar{\theta}_1 = 0.571$ and $\bar{\theta}_2 = 0.6$, the values of Fo_1 and Fo_2 are equal to 0.176 and 0.142, and the numbers z_{01} and z_{02} are equal to 1.19 and 1.21. For $L = l/\xi = 12/5.7 = 2.1$ and $\beta = 3.17 \cdot 10^{-4}$ m/sec$^{1/2}$, the thermal diffusivities are $a_1 = 1.77 \cdot 10^{-8}$ m^2/sec and $a_2 = 1.72 \cdot 10^{-8}$ m^2/sec.

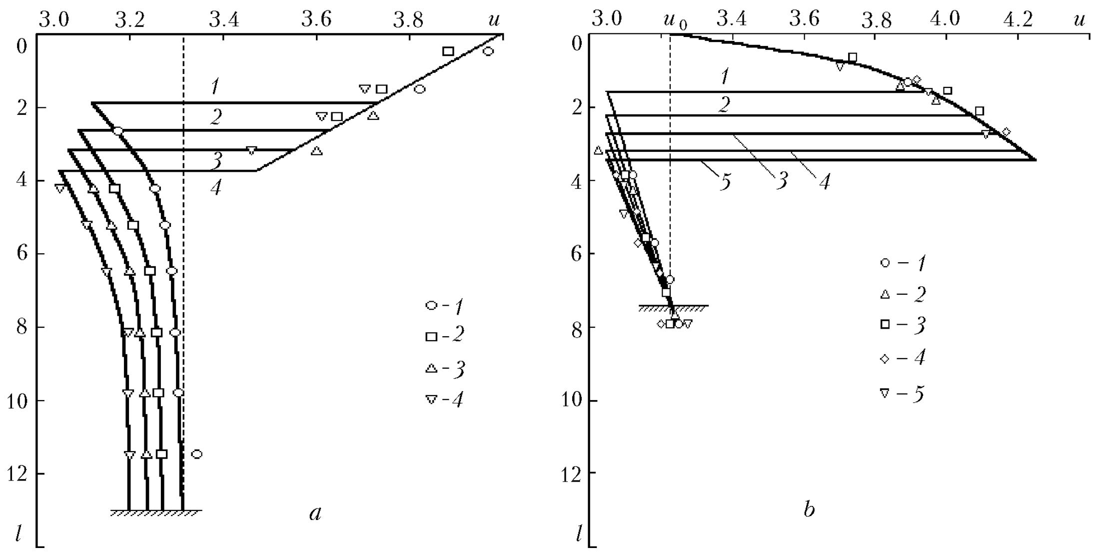

Fig. 4. Change in the moisture content in peat columns at different instants of time τ: a) in the presence of moisture insulation for 2.5 (1), 5.5 (2), 7 (3), and 8.4 h (4); b) in contact with water for 2 (1), 4 (2), 6 (3), 8 (4), and 10 h (5); the temperature of the cooled end of the sample is T_m = 268 K; u = 3.32 kg/kg.

In what follows, we restrict ourselves to consideration of the moisture-content distributions in the frozen and melt zones. For this purpose we must solve Eq. (19) with initial and boundary conditions (20) and (21) and $\partial u(l, \tau)/\partial x = 0$. These solutions are known [1]:

$$U_{\text{melt}} = (u\,(x, \tau) - u^*_{\text{melt}})/(u_0 - u^*_{\text{melt}}) = 2 \sum_{n=1}^{\infty} (-1)^{n+1} \mu_n^{-1} \cos\,(\mu_n x/(l - \xi')) \exp\,(-\mu_n^2 \text{Fo})\,, \tag{27}$$

$$\overline{U}_{\text{melt}} = (\overline{u}\,(\tau) - u^*_{\text{melt}})/(u_0 - u^*_{\text{melt}}) = 2 \sum_{n=1}^{\infty} \mu_n^{-2} \exp\,(-\mu_n^2 \text{Fo})\,. \tag{28}$$

In the frozen zone, we have $\overline{u}_f(\tau) = u_0 + (u_0 - u_{\text{melt}}(\tau))(L - 1)$ or $\overline{U}_f = (u_f(\tau) - u^*_{\text{melt}})/(u_0 - u^*_{\text{melt}}) = 1 + (1 - \overline{U}_{\text{melt}})(L - 1)$. From the latter equalities it follows that the simplex $\overline{U}_{\text{melt}}$ tends to 1 with increase in τ; $\xi \to l$, $L \to 1$, and the moisture accumulation in the frozen zone is $\overline{u}_f \to u_0$ (Fig. 4a). At $\overline{U}_{\text{melt}} = 0.124$, $F_0 = -(4/\pi^2) \ln\,(\pi^2 \overline{U}_{\text{melt}}/8) = 0.76$. Then accordingly we have z_0 = 0.47 and D_{melt} = $1.14 \cdot 10^{-7}$ m^2/sec ($L' = l/\xi'$ = 12/5.4 = 2.22) and the simplex $\overline{U}_f$ is equal to 2.07 and $\overline{u}_f$ is equal to 8.8 kg/kg.

From the experimental values of the distribution of the moisture content over the length of the column moisture-insulated at the bottom (Fig. 4a), we determined the average moisture contents in the frozen $\overline{u}_f$ and melt $\overline{u}_{\text{melt}}$ zones, the Fo numbers, the numbers z_0, and the coefficients of diffusion of moisture in the melt zone D_{melt} for $\beta = 1.9 \cdot 10^{-4}$ m/sec$^{1/2}$ (Table 2).

In the presence of the additional feeding of moisture via the lower base of the column (Fig. 4b), the moisture content of the sample's lower layer remained constant u_0 but the moisture from the feeding layer was transferred in a capillary-osmotic manner through the sample to the zone of phase transition. To take account of this capillary-osmotic transfer we must use Eq. (23) for $x = \xi$ and the solution (14) for the simplex of moisture content $U = (u(x, \tau) - u^*_{\text{melt}})/(u_0 - u^*_{\text{melt}})$. After differentiation and transformations, we have

TABLE 2. Change in the Parameters of Transfer of Moisture (insulated column)

τ, h	ξ, cm	$\bar{u}_f$	u^*_{melt}	$\bar{u}_{melt}$	U	$L-1$	Fo	z_0	$D_{melt}\cdot 10^{-7}$, m/sec^2	u_f	
										experiment	calculation
2.5	1.9	3.87	3.12	3.23	0.550	6.3	0.158	0.2	2.25	3.8	3.9
4.0	2.6	3.81	3.09	3.20	0.478	4.62	0.214	0.234	1.65	3.7	3.8
5.5	2.9	3.77	3.07	3.19	0.469	4.03	0.222	0.263	1.30	3.6	3.7
7.0	3.3	3.74	3.06	3.18	0.462	3.17	0.228	0.330	0.83	3.6	3.6
8.5	3.6	3.71	3.05	3.17	0.444	2.56	0.243	0.346	0.58	3.5	3.6

$$U_f(\tau) = (u_f(\tau) - u^*_{melt})/(u_0 - u^*_{melt}) = 2\text{Fo}(L-1)\left[1 + 2\sum_{n=1}^{\infty} \exp(-(n\pi)^2 \text{Fo})\right]. \tag{29}$$

The average moisture content over the thickness ξ of the frozen layer is

$$\bar{U}_f = (\bar{u}_f - u^*_{melt})/(u_0 - u^*_{melt}) = (L-1)\left[\text{Fo} + \frac{1}{3} - 2\sum_{n=1}^{\infty} (n\pi)^{-2} \exp[-(n\pi)^{-2}\text{Fo}]\right]. \tag{30}$$

According to relation (29), the values of the simplex U_f and accordingly u_f increase in the initial stage of the process of freezing, but then the rate of increase (caused by the phase transition) of the moisture content decreases for $L \to 1$. A linear distribution of the moisture content is observed in the melt zone when the experimental time is long (Fig. 4b), which is consistent with Eq. (14).

When $\text{Fo} > 0.4$ we can disregard all the terms of the series in relations (29) and (30) with an error lower than 4%. Table 3 gives results of the calculations of the numbers $\text{Fo} = U_f/[2(L-1)]$, z_0, and the diffusion coefficients D_{melt} for $\beta = 2.07\cdot 10^{-4}$ m/sec$^{1/2}$.

The calculated data obtained are close to the experimental results. The variants are within the accuracy of determination of moisture contents in the frozen and melt zones. Consequently, formula (30) can be used for prediction of moisture accumulation of a frozen zone in contact of a melt layer of finite thickness with a moisture-saturated ground.

Crystals of ice are formed in the macropores of the ground in which there are a fairly large number of free water molecules. Inside the particles of the ground, the formation of crystals is improbable since water is bound in them in an adsorptive and osmotic manner. This moisture in the vapor and liquid phases can move into the macropores to the ice crystals and can condense on their surface [11]. The process of mass transfer is extended.

Most of the capillary-osmotic moisture becomes ice in a certain zone (see Fig. 1, curve BC), as the temperature changes by 0.4 to 0.5 K (Fig. 2) below the temperature of ice formation of pure water. According to the calorimetric experiments, for different types of peat grounds having an ambiguous bond energy with the material, this range is within fractions of degrees to several degrees. The amount of nonfrozen water decreases with temperature.

As the experiments with a radioactive tag have shown (Fig. 2, curve 3), the migration of moisture in the liquid phase is not observed above the zone of phase transition (see Fig. 1, curve AB). Its low-intensity transfer in the vapor phase is possible.

The sorbed and osmotic moisture inside the ground particles has a higher bond energy with the material than capillary water in the macropores. It is equal to the energy of sublimation of a vapor from the surface of the ice crystals. This is the reason why the co-existence of the liquid and solid phases of water is established in the material at temperatures lower than 273 K [15]. As the ice is accumulated, we have swelling of the peat in the frozen zone and its shrinkage in the melt zone. According to the experiments, the swelling amounts to several percent of the initial thickness of the ground layer.

In the columns of the material under study, capillary water in the melt zone is transferred as a single continuous flow from the base of the column to the zone of ice formation. The intensity of moisture transfer depends on the

TABLE 3. Change in the Parameters of Transfer of Moisture (the column is additionally fed with water)

τ, h	ξ, cm	$L-1$	u_f	Fo	z_0	$D_{melt}\cdot 10^{-7}$, m/sec^2	$\bar{u}_f$	
							experiment	calculation
2	1.6	3.69	3.95	0.72	0.16	4.2	3.7	3.7
4	2.3	2.26	4.07	1.33	0.19	3.0	3.8	3.7
6	2.8	1.68	4.15	1.93	0.21	2.4	3.8	3.7
8	3.2	1.34	4.20	2.52	0.23	2.0	3.9	3.7
10	3.5	1.14	4.25	3.10	0.25	1.7	3.9	3.7

temperature of the medium T_m, the moisture content and structure of peat, the height of the sample, and the absence or presence of additional feeding of moisture at the bottom.

The intensity and accordingly the coefficients of transfer of heat and moisture substantially change with change in the mechanisms of heat and mass transfer. The thermal conductivity $\lambda = ac_e\gamma_0$ depends on the effective specific heat c_e and the density of the solid components (skeleton) of peat γ_0. The effective specific heat is equal to $c_e = c_c + c_i(u - u_{non}) + c_{non}u_{non} + q_i\Delta u/\Delta T$. The component $q_i\Delta u/\Delta T$ allows for the fraction of heat necessary for the phase transition of water to ice. The effective heat capacity c_e is not a constant and depends on the temperature in both the frozen and melt zones. The change in the effective heat capacity c_e for the moisture content of peat $u > 1$ in the zone of phase transition is of particular significance. The ratio c_e/c_c can attain values higher than 30 [11], which causes a corresponding decrease in the thermal diffusivities $a = \lambda/(c_e\gamma_0)$ in the frozen zone.

Adsorbed and osmotic moisture is predominantly present in the sample of highmoor peat (Fig. 2) with an initial moisture content of 2.8. Therefore, the diffusion coefficient of moisture in the melt zone is small (D_{melt} = $0.59\cdot 10^{-8}$ m^2/sec). In a column smaller in length and with an initial moisture content of 3.2, the moisture transfer depends on the time of the process of freezing. At first, the intensity of moisture transfer is high (diffusion coefficient D_{melt} = $2.25\cdot 10^{-7}$ m^2/sec). With drying of the melt zone over a period of 8.5 h, the intensity of transfer to the frozen zone decreases (D_{melt} = $5.8\cdot 10^{-8}$ m^2/sec).

The reason for the decrease in the rate of moisture transfer is the change in the migration mechanism. In the first hours of ice formation, moisture is sucked from the macropores of the entire volume of the sample. In the subsequent time, shrinkage results in the squeezing of water out of the peat particles and its migration to the zone of ice formation. The processes of shrinkage and accordingly squeezing of moisture out of the particles of the material are extended and require considerable capillary-osmotic pressures.

From an analysis of Table 2 it follows that the values of the diffusion coefficients D_{melt} in the process of freezing for 2.5–8.5 h decrease 3.9 times, whereas the numbers z_0 increase 1.7 times.

In the sample of lowmoor peat with an initial moisture content of 7.25 kg/kg (Fig. 3), mass transfer is comparable to the transfer of moisture in a sample with a lower moisture content (Fig. 4a). The experiment was carried out with a closed column, which made the swelling of the sample more difficult.

An intense moisture accumulation in the frozen layer occurs in the sample with additional feeding (Fig. 4b). The moisture accumulation increases with freezing due to the transit capillary transfer in the macropore system of the melt zone from the moisture-saturated layer lying below. Moisture arrives at the frozen layer first from the melt zone and the moisture-saturated layer. But with freezing, the fraction of the arrival of moisture from the zone of additional feeding increases because of the decrease in the melt zone itself. That is why the coefficients of diffusion of moisture in the melt zone D_{melt} decrease 2.5 times in the process of freezing, whereas the numbers z_0 increase 1.6 times (Table 3). The diffusion coefficients are 2 to 3 times larger in the presence of additional feeding than in the absence of it.

From an analysis of the experimental and calculated results given in the paper it follows that it is more preferable to select the z_{01}, z_{02}, and z_0 numbers than the transfer coefficients. Their predetermination on identical samples or on those close in thermophysical and structural properties will enable one to obtain the distributions (consistent with experiment) of the temperature and the moisture content in the frozen and melt zones of similar actual grounds.

NOTATION

a_i, thermal diffusivities of the frozen and melt zones, m^2/sec; $a(x)$, $b(x)$, and $c(x)$, coefficients of Eq. (1); c_e, effective specific heat, J/(kg·K); c_c, c_i, and c_{non}, specific heats of the solid components of peat, ice, and nonfrozen water, J/(kg·K); D_{melt}, coefficient of diffusion of moisture in the melt zone, m^2/sec; $F(\rho, \tau) = \int_{y_1}^{y_2} f(x, \tau)K(x, \rho)dx$, transform of the function $f(x, \tau)$ in Eq. (1); $\overline{F}(\rho, \tau) = \int_0^{\tau} \exp(\rho^2\tau) \int_{y_1}^{y_2} f(x, \tau)K(x, \rho)dxd\tau$, integral of the function $F(\rho, \tau)$; $Fo_1 = a_1\tau/\xi^2$ and $Fo_2 = a_2\tau/(l - \xi)^2$, Fourier numbers; $H(x, \rho)$ and $H_j(x, \rho)$, functions taking account of the boundary conditions of the boundary-value problem and their values for $x = y_j$ $(j = 1, 2)$, $j \in n$; $K(x, \rho)$, kernel of the integral transformation; l, thickness of the layer (sample), m; $L = \xi/l$, ratio of the depth of freezing to the thickness of the ground layer; $M(\rho)$, normalizing divisor; $N(x, \rho)$, eigenfunctions of the Sturm–Liouville problem; n, numbers of the natural series; $p(x) = a(x)\sigma(x)$; $q(x) = -c(x)\sigma(x)$; q_i, heat of phase transition, J/kg; R and R_0, time-variable and initial radioactivities, pulse/min; T, T_0, T_m, T_{ph}, and $\overline{T}_j$, running, initial, ambient-medium, phase-transition, and average-over-the-layer thickness temperatures, K; u, u_0, u_{melt}, u_m, and u^*_{melt}, variable and initial moisture contents and moisture contents in the melt and frozen zones and at the boundary of these zones, kg/kg; u_{non}, amount of nonfrozen water, kg/kg; $\overline{u}$, moisture-content average over the zone thickness, kg/kg; $w(x)$ and $w_0(x)$, function of the temperature or the moisture content in Eq. (1) and its initial distribution in the limits $y_{10} \le x \le y_{20}$; $\overline{w}(\rho, \tau) = \int_{y_1}^{y_2} w(x, \tau)K(x, \rho)dx$ and $\overline{w}(\rho, 0) = \int_{y_1}^{y_2} w_0(x)\Big|_{x=y_{10}}^{x=y_{20}} K(x, \rho)dx$, transforms of the functions $w(x)$ and $w_0(x)$; x, coordinate, m; $y(\tau)$, thickness of freezing of the ground layer at the instant of time τ; y_1, y_2, and y_{01}, y_{02}, coordinates of the layer boundaries and their initial values, m; x' and y', first derivatives; $z_{0i} = \beta/(2a_i^{1/2})$, numbers for the frozen ($i = 1$) and melt ($i = 2$) zones; $\beta = \xi/\tau^{1/2}$, coefficient, m/sec$^{1/2}$; $\gamma_0 = \gamma/(1 + u)$; γ, density of the ground, kg/m^3; $\overline{\theta}_1 = (T_1(\tau) - T_m)/(T_{ph} - T_m)$ and $\overline{\theta}_2 = (T_2(\tau) - T_{ph})/(T_0 - T_{ph})$, simplexes; $\mu_n = (2n - 1)\pi/2$; ξ and ξ', coordinates corresponding to the lower boundary at T = 273 K and to the center of the freezing zone (Fig. 3), m; ρ, eigenvalues of the Sturm–Liouville problem; $\sigma(x)$, weight function; τ, time, sec. Subscripts: 0, initial value; c, component; i, ice; f, frozen; non, nonfrozen; m, medium; melt, melt; ph, phase transition; e, effective.

REFERENCES

1. A. V. Luikov, *Heat-Conduction Theory* [in Russian], Vysshaya Shkola, Moscow (1967).
2. É. M. Kartashov, *Analytical Methods in Heat Conduction of Solids* [in Russian], Vysshaya Shkola, Moscow (1985).
3. G. A. Grinberg, A possible approach to consideration of the problems of the theory of heat conduction and to diffusion, wave, and similar problems in the presence of moving boundaries and some other applications of it, *Prikl. Mat. Mekh.*, **31**, Issue 2, 193–203 (1961).
4. G. A. Grinberg and O. M. Chekmareva, Motion of the phase interface in Stefan-type problems, *Zh. Tekh. Fiz.*, **60**, No. 4, 2025–2031 (1970).
5. N. S. Ivanov, *Heat and Mass Transfer in Frozen Rocks* [in Russian], Nauka, Moscow (1963).
6. V. G. Melamed, *Heat and Mass Transfer in Rocks in Phase Transitions* [in Russian], Nauka, Moscow (1980).

7. G. M. Fel'dman, *Methods of Calculation of the Temperature Mode in Frozen Grounds* [in Russian], Nauka, Moscow (1973).
8. S. E. Grechishchev, L. V. Chistotinov, and Yu. B. Shur, *Cryogenic Physico-Geological Processes and Prediction of Them* [in Russian], Nauka, Moscow (1980).
9. A. V. Pavlov, *Calculation and Control over the Congelation Mode of the Ground* [in Russian], Nauka, Siberian Branch, Novosibirsk (1980).
10. G. P. Brovka, *Heat and Mass Transfer in Natural Disperse Systems in Freezing* [in Russian], Nauka i Tekhnika, Minsk (1991).
11. N. I. Gamayunov, V. A. Mironov, and S. N. Gamayunov, *Heat and Mass Transfer in Organogenic Materials. Cryogenic Processes* [in Russian], TGTU, Tver' (1998).
12. N. I. Gamayunov, Heat and moisture transfer in swollen grounds, *Pochvovedenie*, No. 11, 1330–1336 (1996).
13. A. M. Globus, *Physics of Nonisothermal Intraground Moisture Exchange* [in Russian], Gidrometeoizdat, Leningrad (1983).
14. N. S. Koshlyakov, É. B. Gliner, and M. M. Smirnov, *Partial Differential Equations of Mathematical Physics* [in Russian], Vysshaya Shkola, Moscow (1970).
15. N. I. Gamayunov and S. N. Gamayunov, Sublimation drying of peat, *Inzh.-Fiz. Zh.*, **77**, No. 5, 67–71 (2004).